Genetic-Based Subcarrier and Bit Allocation Algorithm for Multiuser OFDM System

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Abstract

This paper proposes an adaptive subcarrier and bit allocation technique for multiuser OFDM (Orthogonal Frequency Division Multiplexing) system based on genetic algorithm. The objective is to minimize the overall transmission power while ensuring the per user's data rate and Bit Error Rate (BER) requirements. This has been achieved by developing a search algorithm based on genetic system. Simulation results show that the performance of the proposed algorithm outperforms the Wong's adaptive OFDM (MAO) algorithm in terms of computational complexity.

Keywords

OFDM, Multiuser, Subcarrier, Bit, Allocation, Genetic.

1. Introduction

It has been suggested that multiuser OFDM systems employing adaptive subcarrier and bit allocation can take advantage of the channel diversity among users in different locations, thereby enabling an efficient use of all subcarriers in the OFDM system. Optimal bit loading and subcarrier-allocations have already been formulated by, for example, Wong in (C. Y. Wong, et al., 1999) , (W. Rhee and J. M. Cioffi, 2002): more specifically, by minimizing the overall transmission power under a given data rate constraint in (C. Y. Wong, et al., 1999) , and by maximizing the data rate under a given power constraint in (W. Rhee and J. M. Cioffi, 2002). These are both nonlinear optimization problems with integer variables and are referred to as "marginal adaptive" (MA) and "rate adaptive" (RA) optimizations in (T. Starr et al., 1999). However, solving these problems is extremely difficult; therefore, they are only solved by relaxing the integer variables requirements and allowing real numbers. Consequently, even though this approach requires an intensive computation, it cannot yield an optimal solution. To reduce the computational complexity, some suboptimal algorithms have been proposed for MA and RA optimizations in (C. Y. Wonget al., 1999)–(Y. J. Zhang and K. B. Letaief, 2004) . Even though, the use of these algorithms still rather limited for the following reasons: In (C. Y. Wonget al., 1999), it is assumed that the average signal-to-noise ratios (SNRs) for all users are identical; while in (Y. J. Zhang and K. B. Letaief, 2004) , an equally distributed transmission power among all subcarriers is used.

This paper shows that the above nonlinear optimization problem can be formulated, and solved using genetic algorithm. Simulation results have confirmed that our approach outperforms that in (C. Y. Wong, et al., 1999) with less computational complexity, hence it provides a better ability to be used in conjunction with real-time applications.

The rest of this paper is organized as follows. In section 2, the multiuser OFDM system model is described, and the dynamic resource allocation problem is formulated. The genetic-based allocation algorithm is developed in section 3. Numerical results are presented and discussed in section 4. Finally, conclusions are drawn in section 5.

2. System model

The structure of the adaptive multiuser OFDM system under consideration is shown in Figure 1. The system has K users and use N subcarriers. The base station receives (downlink) channel state information from all users via some feedback signaling system which we assume existing. The base station uses this information to assign a set of subcarriers to each user and determines the number of bits per OFDM symbol to be transmitted by each subcarrier. It is assumed that sharing a subcarrier by two or more users is not allowed. Depending on the number of bits assigned to each subcarrier, an adaptive modulation scheme is elected per subcarrier. The subcarrier and bit allocation information are sent to the receivers via a separate signaling channel.

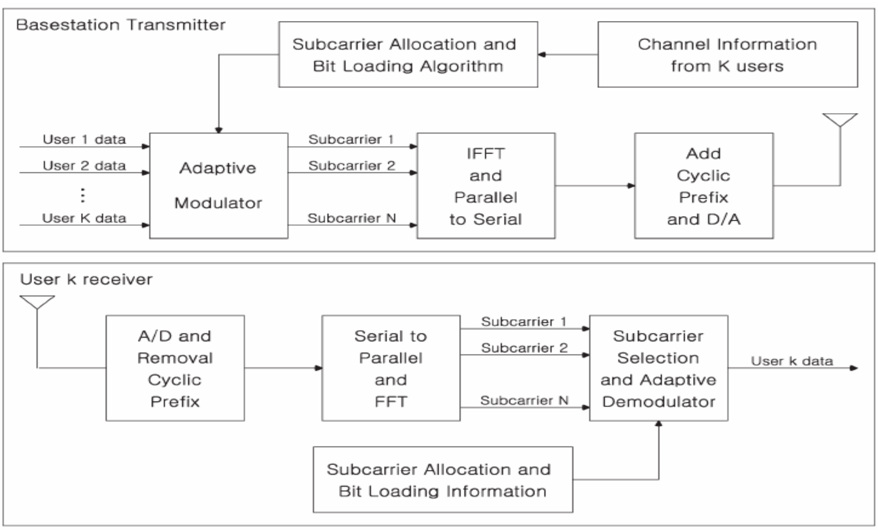

Figure 1: Adaptive multiuser OFDM system.

At each receiver, the subcarriers assigned to each user are selected, and the signal associated with each subcarrier is demodulated. In the following, the bit loading and subcarrier-allocation is formulated as a constraint optimization problem.

Let R_k denotes the data rate of the *k*th user, $c_{k,n} \in D$, number of bits assigned to the *n*th subcarrier of *k*th user where D is a set of nonnegative integers that are less than or equal to M, and M is the maximum number of bits/symbol that can be transmitted by each subcarrier. Therefore, the data rate, R_k, can be expressed as $R_k = \sum_{n=1}^{N} C_{k,n}$ or equivalently:

$$R_k = \sum_{n=1}^{N} c_{k,n}\rho_{k,n}, \tag{1}$$

Where $\rho_{k,n}$ is an indicator variable defined as,

$$\rho_{k,n} = \begin{cases} 1, & if \quad c_{k,n} \neq 0 \\ 0, & Otherwise \end{cases}, \tag{2}$$

and $\sum_{k=1}^{K} \rho_{k,n} = 1$ because a subcarrier can be occupied by at most one user. The transmission power allocated to the nth subcarrier of user k is expressed as: (C. Y. Wong, et al., 1999)

$$P_{k,n} = \frac{f_k(c_{k,n})}{\alpha_{k,n}^2} \tag{3}$$

where $\alpha_{k,n}$ is the channel gain between the base station and user k on the nth subcarrier, and $f_k(c_{k,n})$ is the power required for a reliable reception of $c_{k,n}$ bits/subcarrier when the channel gain is equal to unity. Assuming that M-QAM modulation is being used. Therefore, $f_k(c)$ is given by (C. Y. Wong, et al., 1999):

$$f_k(c) = \frac{N_0}{3}\left[Q^{-1}\left(\frac{BER_k}{4}\right)\right]^2 (2^c - 1) \tag{4}$$

where *No/*2 is the variance of the AWGN, and *Q*(*x*) denotes the *Q*-function (K. Zhou et al., 2005). The subcarrier, and bit allocation problem such that the total transmission power is minimized under a given data rate and BER constrains, can be formulated as follows:

$$\min_{c_{k,n},\rho_{k,n}} P_T = \min_{c_{k,n},\rho_{k,n}} \sum_{k=1}^{K}\sum_{n=1}^{N} \frac{f_k(c_{k,n})}{\alpha_{k,n}^2}.\rho_{k,n} \tag{5}$$

Subject to:

$$R_k = \sum_{n=1}^{N} c_{k,n}\rho_{k,n} \quad \forall k, \tag{6}$$

$$\sum_{k=1}^{K} \rho_{k,n} = 1 \quad \forall n, \tag{7}$$

$$BER_k \le BER_{t\arg et} \quad \forall k \tag{8}$$

where $BER_{t\arg et}$ denotes the target bit error rate required for user k .

3. Genetic-based multiuser subcarrier and bit allocation algorithm

Genetic algorithm can be used to solve a variety of optimization problems that are not well suited for standard optimization systems. This include problems with discontinuous objective functions (non-differentiable) and/or highly nonlinear. In a genetic based optimization problem, a population is generated at random, modified according to some rules in order to obtain children. Based on the modification rules, the population evolves successively towards an optimal solution. In general genetic algorithm uses three main types of rules at each step to create the next generation from a given current population:

- Selection rules: select the individuals (called parents) that contribute to the population at the next generation.
- Crossover rules: combine two parents to form children for the next generation.
- Mutation rules: apply random changes to individual parents to form children.

The following section presents the genetic-based multiuser subcarrier and bit allocation algorithm developed in this paper which is divided into a number of steps as follows:

Step 1: Subcarrier and bit allocation (SBA):

First define an array with $N+K$, elements. This array can be represented as the union of two arrays as follows:

Sub#1	Sub#2	…	Sub #N	User #1	User #2	…	User #K

Figure 2: Subcarrier and Bit Allocation (SBA) array as coded in genetic algorithm.

a. The first array is called the Subscriber Allocation (SA) array which it consists of N -elements with each element represents a subcarrier in the OFDM system as shown in Figure:2 .

The value of each element in the SA array is confined to integers from 1 to K that represents the users. For example, if the first element of the array is l, this means that subcarrier 1 is allocated to user l, $1 \leq l \leq K$. As an example, the subscriber allocation array can take the following form:

$$SA = [K, K-2, K, ..., K-1, K, 1, K-2, ...] \tag{9}$$

b. Second array is called the Bit Allocation (BA) array which it consists of K -elements with each element represents a user in the OFDM system as shown in Figure: 2.

The value of each element in the BA array is confined to integers from 0 to M that represent the bit loading for each user. For example if the first element of the array is c, this means that the subcarriers allocated to user #1 are bit loaded by c -bits per subcarrier, $0 \leq c \leq M$.

As an example, the bit allocation array can take the following form:

$$BA = [c_1, c_2, c_3, ..., c_K] \tag{10}$$

Therefore, the subcarrier and Bit Allocation (SBA) array can take the following form,

$$SBA = SA \cup BA = [K, K-2, K, ..., K-1, K, 1, K-2, ... \mid c_1, c_2, c_3, ..., c_K] \tag{11}$$

The genetic algorithm starts by generating SBA array with random elements and eventually it provides the optimal solution.

Step 2: Generation of population (POP):

At the beginning, an initial population array is randomly generated.

Step 3: Evaluation phase:

For each subcarrier and bit allocation represented by a given SBA array, the following objective function is used as a fitness measure of this array:

$$F = \sum_{k=1}^{K}\sum_{n=1}^{N}\frac{f_k(c_{k,n})}{\alpha_{k,n}^2}.\rho_{k,n} + S\left(\sum_{k=1}^{K}\left|\sum_{n=1}^{N}c_{k,n}\rho_{k,n} - R_k\right|\right) = P_T + SE \tag{12}$$

where $E = \sum_{k=1}^{K}\left|\sum_{n=1}^{N}c_{k,n}\rho_{k,n} - R_k\right|$ is defined as an error term, and $f_k(c)$ is given by:

$$f_k(c_{k,n}) = \frac{N_0}{3}\left[Q^{-1}\left(\frac{BER_k}{4}\right)\right]^2(2^{c_{k,n}} - 1) \tag{13}$$

with,

$$\rho_{k,n} = \begin{cases} 1 & k = SBA(n) \\ 0 & Otherwise \end{cases} \tag{14}$$

$$c_{k,n} = \begin{cases} SBA(N+k) & \forall n \in T_k \\ 0 & Otherwise \end{cases} \tag{15}$$

where, T_k is the set of subcarrier indexes assigned to the *k*th user as derived from (14), S is a scaling factor used to guarantee that the fitness function (12) for a given $SBA = SBA_{opt}$ is less than for $SBA \neq SBA_{opt}$, where SBA_{opt} is an array representing the optimal solution of (12).

The fitness function in (12) is seen as the sum of two terms which controls the final optimal allocation as follows:

a. The first term is the total transmitted power, it represents the system efficiency: the larger the channel gain, the lower the needed power, and the lower the value of the fitness function. So, in order to find the allocation with the lowest transmission power, the user with the largest channel gain at a given subcarrier will have better chance to use it.
b. The second term (SE) is added to guarantee fairness among users of the OFDM system. By fairness, we mean that for each user, no matter how 'good' or 'bad' his channel gain values (Z. Shen et al., 2005), he should be allocated a certain number of subcarriers so that his data rate is satisfied. Clearly, at optimal subcarrier and bit allocation, the term E must go to zero.

Determination of the scaling factor (S):

When the array $SBA = SBA_{opt}$, then the error term in (12) equal to zero, that is,

$$E = \sum_{k=1}^{K} \left| \sum_{n=1}^{N} c_{k,n}\rho_{k,n} - R_k \right| = 0 \rightarrow \sum_{n=1}^{N} c_{k,n}\rho_{k,n} = R_k \quad \forall k \tag{16}$$

and,
$$F\big|_{SBA=SBA_{opt}} = P_{\min} = \sum_{k=1}^{K}\sum_{n=1}^{N} \frac{f_k(c_{k,n})}{\alpha_{k,n}^2}.\rho_{k,n} \tag{17}$$

Equation (16) shows that each user, k, will take certain number of subcarriers $(\rho_{k,n} \neq 0 \quad \forall n)$ to achieve his data rate requirements. The modulus is taken over the summation term in (16) in order to guarantee that the data rate for each individual user is achieved. When $SBA \neq SBA_{opt}$ then, the error term will not equal to zero,

$$E = \sum_{k=1}^{K} \left| \sum_{n=1}^{N} c_{k,n}\rho_{k,n} - R_k \right| \neq 0 \tag{18}$$

and,
$$\sum_{k=1}^{K}\sum_{n=1}^{N} \frac{f_k(c_{k,n})}{\alpha_{k,n}^2}.\rho_{k,n} = \bar{P} \neq P_{opt} \tag{19}$$

Therefore,
$$F\big|_{SBA \neq SBA_{OPT}} = \bar{P} + SE \tag{20}$$

As mentioned earlier, the value of the fitness function at $SBA = SBA_{opt}$ must be less than the value of the fitness function at $SBA \neq SBA_{opt}$ that is,

$$F\big|_{SBA=SBA_{OPT}} < F\big|_{SBA \neq SBA_{OPT}} \rightarrow P_{opt} < \bar{P} + SE \tag{21}$$

The condition in (21) may not be satisfied for all allocation arrays generated by the genetic algorithm which means that for some SBA, the value of it's fitness function will be smaller than the optimum value and one of these arrays will be selected as the solution of the optimization problem. Clearly, such an "optimal" value does not satisfy the rate constraint for each user.

In order to avoid these circumstances, the scaling factor S must be chosen with a value that satisfy the condition in (21) as follows: (C. Y. Wong, et al., 1999)

$$S > \frac{P_{opt} - \bar{p}}{E} \tag{22}$$

Since E is positive, integer, and $\bar{P}$ is positive, we can choose S as any value greater than P_{opt} ,

$$S > P_{opt} \quad \text{which means,} \quad P_{opt} > \frac{P_{opt} - \bar{p}}{E} \tag{23}$$

but P_{opt} is not known. Therefore, S can be chosen as any value greater than $P_{\max} > P_{opt}$ where $P_{\max}$ is the maximum total power:

$$P_{\max} = \sum_{n=1}^{N} \frac{\left(\frac{N_0}{3} \left[Q^{-1}\left(\frac{BER_{\min}}{4} \right) \right]^2 (2^M - 1) \right)}{\alpha_{\min}^2} \tag{24}$$

Thus, the value of S can be chosen to satisfy the following inequality,

$$S > \sum_{n=1}^{N} \frac{\left(\frac{N_0}{3} \left[Q^{-1}\left(\frac{BER_{\min}}{4} \right) \right]^2 (2^M - 1) \right)}{\alpha_{\min}^2} \tag{25}$$

where $\alpha_{\min} = \min \alpha_{k,n}$ is the minimum channel gain level, and $BER_{\min} = \min BER_k$ is the minimum bit error rate in the OFDM system.

As an example assume that the number of subcarriers in the OFDM system $N = 8$, the number of users $K = 4$, the number of bits transmitted per OFDM symbol for each user $R_k = 4$, and the number of bits of the *k*th user assigned to the *n*th subcarrier $c_{k,n} \in [0,1,2,3,4]$.

Assume that the following subcarrier and bit allocation (SBA) array is generated by genetic algorithm which represents the solution for the optimization problem.

$$SBA_{opt} = [1,2,3,2,4,3,2,2 \mid 4,1,2,4] \tag{26}$$

Then from (14) and (15), we can compute $\rho_{k,n}$ and $c_{k,n}$ as follows:

$$\rho_{k,n} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \qquad c_{k,n} = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \end{bmatrix} \quad (27)$$

from (27), we have:

- Subcarrier #1 is allocated to user #1 and bit loaded with 4-bit to achieve the required data rate $R_1 = 4$ bits/symbol.
- Subcarrier #2,4,7,8 are allocated to use r#2 and bit loaded with 1-bit/subcarrier to achieve the required data rate $R_2 = 4$ bits/symbol.
- Subcarrier #3,6 are allocated to user #3 and bit loaded with 2-bit/subcarrier to achieve the required data rate $R_3 = 4$ bits/symbol.
- Subcarrier #5 are allocated to user #4 and bit loaded with 4-bit to achieve the required data rate $R_4 = 4$ bits/symbol.

Step 4: Selection:

Using the roulette wheel parent selection POP/2, pairs of parents are chosen from the current population to form a new population.

Step 5: Crossover:

With probability of crossover P_c, children are formed by performing crossover on the POP /2 pairs of parents. The children replaces the parents in the new population. Two-point crossover is used in our algorithm.

Step 6: Mutation:

With probability of mutation P_m, mutation is performed on the new

population.

Step 7: New population:

The new population becomes the current population for the new generation steps 4-7 are repeated until the predefined generation number (assume to be G) is reached. The best array in the last population is the solution.

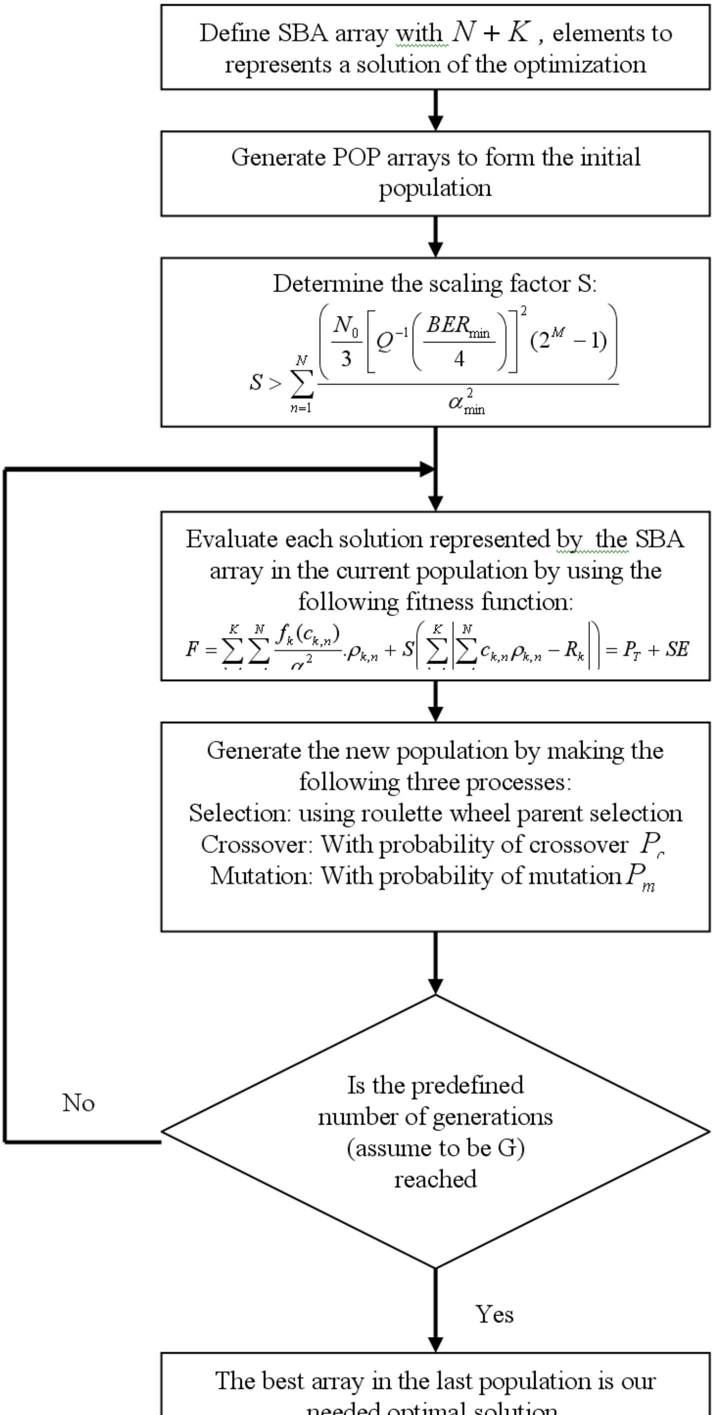

Figure 3: Proposed genetic-based subcarrier and bit allocation chart

4. Numerical results

In this section, we compare the performance of the proposed genetic-based allocation algorithm with Wong's adaptive OFDM (MAO) algorithm (C. Y. Wong, et al., 1999) using self implemented simulation tool. We consider an OFDM system with N=40 subcarriers, K=2 to 10 users. Each user transmits R_k=20bits in an OFDM symbol with target BER set to 10^{-3}. The channel state information at each subcarrier is generated randomly according to a Rayleigh distribution. We also assume that the system adopts the M-ary QAM with D={0,2,4,6,8}. Square signal constellations (4-QAM,16-QAM,64,QAM, and 256-QAM) are used to carry two, four, six, or eight bits/subcarrier. The parameters for the genetic algorithm are listed in table 1 below.

Parameter	Value
Initial population size: POP	100
Probability of crossover: P_c	0.5
Probability of mutation: P_m	0.5
Number of generations: G	100
Scaling factor: S	10000

Table 1: Parameters of genetic based allocation algorithm

The above scenario is used to compare the performance of our genetic-based allocation algorithm with the MAO algorithm in (C. Y. Wong, et al., 1999) . Here, we obtain the overall power, and compare the computational time that needed for both the modified genetic-based allocation and the algorithm in (C. Y. Wong, et al., 1999) under different situations.

Figure:4, and Figure:5, show the overall power and the corresponding computational times needed for:

- Wong's adaptive OFDM (MAO) algorithm (C. Y. Wong, et al., 1999) .
- Genetic-based allocation with constant bit loading overall subcarriers $(c_{k,n} = cons\tan t \quad \forall k, n)$.
- Genetic-based allocation with constant bit loading for each user $(c_{k,n} = cons\tan t \quad \forall n \in T_k)$.
- Genetic-based allocation with adaptive bit loading overall subcarriers.

From Figure:4 and Figure:5 we can see that the genetic-based allocation with adaptive bit loading overall subcarriers outperforms that in (C. Y. Wong, et al., 1999) in terms of needed power but it takes a larger computational time. On the other hand, the genetic based allocation with constant bit loading for each user outperforms the algorithm in (C. Y. Wong, et al., 1999) in terms of computational time and it gives almostly the same power. We also show that the computational time needed in case of constant bit loading overall subcarriers is slightly small than that of constant bit loading for each user but with large needed power.

Figure:6 and Figure:7 depicts the average minimum power in every generation for the genetic-based allocation at different number of users in the following cases:

- Constant bit loading overall subcarriers $(c_{k,n} = constant \quad \forall k, n)$.
- Constant bit loading for each user $(c_{k,n} = constant \quad \forall n \in T_k)$.
- Adaptive bit loading overall subcarriers.

From Figure:6 and Figure:7 we can see that, in the case of *2* users, both algorithms (constant bit loading for each user, and adaptive bit loading overall subcarriers) converge very quickly to the optimal result. Clearly, this is so because the number of users is small and hence, the two algorithms can easily reach to the optimal allocation.

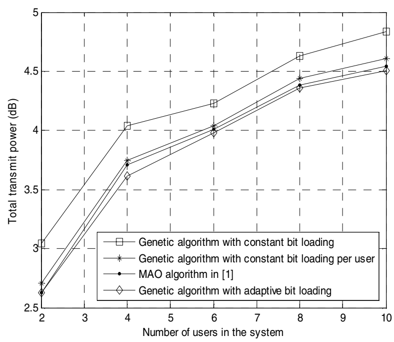

Figure 4: Total transmit power required by different algorithm.

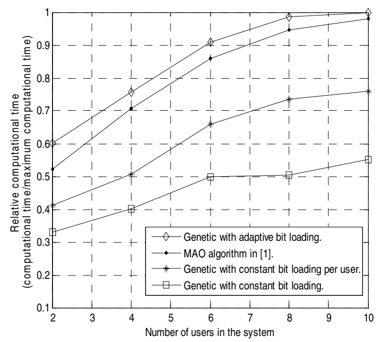

Figure 5: Relative computational time required by different algorithm.

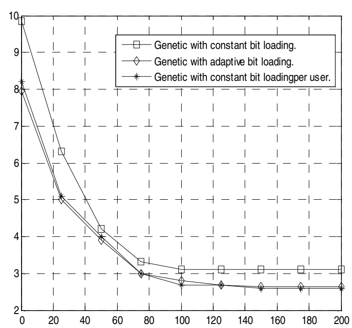

Figure 6: Convergence of genetic-based algorithm in the case of 2-users.

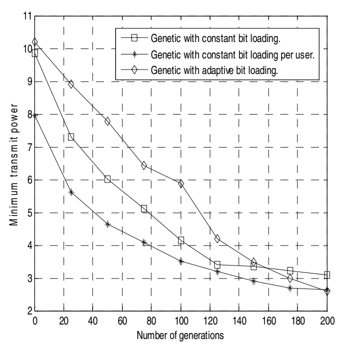

Figure 7: Convergence of genetic-based algorithm in case of 8-users.

However, as the number of users increases, the constant bit loading for each user algorithm converges much more quickly than that for the adaptive bit loading. From the simulation results, we can conclude that the proposed genetic-based allocation algorithm with constant bit loading for each user is a better allocation scheme than the MAO algorithm (C. Y. Wong, et al., 1999) as it converges quickly (small computational time) and gives almost the same needed power.

5. Conclusion

This paper, proposes a genetic-based algorithm for the subcarrier and bit allocation in multiuser OFDM system. The algorithm dynamically allocates subcarriers and bits for users in the system by monitoring the instantaneous channel gain. Without degradation of performance, the proposed algorithm reduces the computational complexity of the nonlinear optimization used by Wong's adaptive OFDM (MAO) algorithm (C. Y. Wong, et al., 1999) . The results have shown that the genetic based algorithm with constant bit loading for each user can be used to provide the same performance as in Wong's MAO algorithm but with less computation complexity.

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