COMMUNICATION NETWORK OPTIMISATION: PAST, PRESENT AND FUTURE.

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ABSTRACT.

This paper surveys the major facets of the branch of combinatorial optimisation known as communication network optimisation. The basic principles are investigated briefly, followed by a discussion of location, connection and traffic routeing problems in turn. In each case the relevant existing algorithms are listed together with some suggestions concerning the likely areas of investigation in future years.

1. INTRODUCTION.

It is common for communication networks to make use of the most modern equipment available. Such equipment is, by definition, expensive and it clearly becomes important to use it in the most efficient way possible. It is this pursuit of perfection, in its various forms, which comprise the field of communication network optimisation.

There are a number of individual problems to be dealt with in the design of any particular network. Each may or may not be appropriate in any given situation. This paper considers the more important of these by presenting the major pieces of work which have been carried out in each case. Before this can be achieved, however, some basic ground rules are necessary.

1.1. Objectives.

Any variable system can only be optimised with respect to a single objective function although this may be a non-trivial combination of a number of components. The optimisation discussed in this paper will be mainly carried out with respect to minimising the cost of the resultant network. In general, a number of constraints will be stipulated and the network of minimum cost, satisfying these constraints, will be sought.

There are alternative forms of optimisation, however. If a predetermined amount of facilities are available then the cost can be considered as one of the fixed constraints and different parameters such as reliability (Hansler et al. [1972], Van Slyke & Frank [1972] and Wilkov [1972]), grade of service (Gunnarsson & Nivert [1984]), congestion (Davies [1972]) or traffic flow can be maximised or minimised as appropriate.

1.2. Simple Principles of Communication Networks.

The purpose of any network is to connect together a number of sources and sinks of communications traffic, known as nodes, distributed within a given region. The nodes are situated in such a way as to serve the underlying traffic distribution of the region. If the locations of the nodes are known then so is the amount of traffic flowing in each direction between each node pair. The number of nodes is commonly represented by n.

A solution to a network problem will consist of a system of links and switches (sometimes known as tandems), collecting and distributing the traffic around the network in the required manner. Both the topological structure of this network and the way in which the traffic flows around it are subjects for optimisation. There is clearly a strong relationship between these networks and the mathematical subject of graph theory (Harary [1971] and Wilson [1972]). This relationship is investigated in Cattermole [1975] and Grout [1988].

The cost of any network will be the sum of the individual costs of its component links, nodes and switches. The cost of a link, node or switch is dictated by its size which, in turn, depends on the amount of traffic it has to deal with (and in the case of a link, its length). For these purposes, approximations such as the Erlang B formula are generally used (Hills [1979]). It is the purpose of this paper to survey the principal techniques for optimisation available and not to deal, in any great detail, with the mathematical technicalities. For such a treatment, see Meister et al. [1972] or Grout [1988].

2. LOCATION OF FACILITIES.

The effective placement of nodes and switches will have a considerable influence upon the cost of the resultant network. Ideally, each should be situated in the location where it most efficiently collects (and distributes) the traffic which it has to deal with. This section deals with nodes and switches in turn.

2.1. Traffic Server Locations.

For any given region there will be a certain distribution of traffic generated and received. The nodes should be placed in such a way as to minimise the cost of serving this distribution. There are two problems to be dealt with: that of determining the number of nodes (and consequently the area covered by each node) and the location of each node within its respective area.

One of the first real attempts to deal with these problems is given in Rapp [1962]. It is noted that the cost of a node within a single traffic area is a function $F(x,y,D)$ of its x and y coordinates and the traffic distribution D. The optimum location of the node is then given by $\partial F/\partial x = \partial F/\partial y = 0$. The size of the node should then be calculated merely to be large enough to deal with the traffic in that area. The traffic distribution is typically represented by a numerical grid (Rapp [1962], Enriquez de Salamanca & Zulueta [1971] and Wadhwa [1979]) with larger values representing areas of greater traffic density. This situation is shown in Fig. 1.

In practice the node placement can be achieved with reasonable accuracy (bearing in mind that traffic density figures are likely to be imprecise anyway) by balancing moments or, even more simply, by ensuring that the traffic totals on either side of an imaginary straight line drawn through the node are equal. Similar work has been carried out by Okazaki [1982].

The situation is not, in truth, as simple as this. The ultimate objective in network design is to minimise the total cost. Placing the node in such a way as to minimise the cost of the subscriber connections in that group may cause the links between the node and its parent tandem(s) to be longer than necessary. If the cost of these links is considered, the effect is to pull the node away from the centre of the group (see Fig. 2). Taking this into account, the problem of node placement is more difficult and may be a productive area of investigation for the future.

2.2. Switching Centre Locations.

If the positions of the n nodes are established it then becomes necessary to determine the optimum location of the M switches in the higher level network as well as the best value of M. A number of constraints are possible at this stage, depending upon the form of the solution required.

Hoshi [1985] addresses the problem of finding the optimum size of each tandem group. Clearly the mean size of each group is inversely proportional to the number of tandems. A number of somewhat simplistic assumptions are made in this paper but it does provide an explicit method for determining M.

The general problem of determining the best choice of tandems has been considered by Bahl & Tang [1972], Boorstyn & Frank [1977], Tang et al. [1978], Hoc [1982], Mirzaian [1985] and Grout et al. [1986,1987] among many others. The approaches are many and varied. Some algorithms only work for networks of a specific type whereas others tend to make impractical assumptions such as knowing the cost of variable amounts of equipment in advance or having it proportional to a simple formula (see Grout [1988] for a fuller discussion of the shortcomings of some existing methods).

The general problem is extremely difficult. To know the cost of a tandem requires that its size be known also. This will depend on the closeness of the other tandems which, in turn, implies prior knowledge of the tandem locations, the very thing required at the beginning. The only escape is to consider each selection of tandems on its merits, one at a time. To examine every tandem set in this way, however, would take too long and, in practice, heuristic techniques such as the Double Drop method (BP [1984]) or that of Grout et al. [1986,1987] are used instead. These methods do not guarantee the exact optimum solution but are found, in practice, to provide approximations of acceptable accuracy. The Double Drop method begins with all nodes established as switches and systematically removes them until the required set is reached. The method of Grout et al. employs a reduction stage to simplify the problem followed by a constrained technique for tandem selection.

The most profitable direction for research in the future would be into ways of improving these practical heuristics, both in terms of speed of execution and accuracy of results. The continually improved computational power becoming available (Sumner [1986]) would appear to ensure the former to some extent.

3. NETWORK CONNECTION TOPOLOGIES.

With the position of the n nodes and M switches established, the problem becomes one of connection rather than location. There are at least as many algorithms for the connection problem as for the location problem (indeed some methods deal with both simultaneously), each varying according to the particular application it is intended to deal with. Firstly, a special type of network is considered, followed by the more general case.

3.1. Centralised Networks.

These networks are those of the form discussed by Chandy & Russell [1972] and correspond to systems where communication is only required between a number of terminals and a central computer or data base (see Fig. 3). A number of algorithms exist for their solution. Most are based, in some way or another, on tree structures (Kruskal [1956] and Prim [1957]). The minimal spanning tree (MST) of a set of nodes can be computed easily (Shamos [1978]). If the MST were always suitable the problem would be solved. However practical requirements of link capacity and reliability usually imply a series of constraints upon the final solution which complicate the situation.

Exhaustive search techniques are not desirable if the number of nodes is large. Chandy and Russell [1972] present a method for determining the optimum network based on the branch and bound work of Little et al. [1963]. This is still, in effect, merely a structured exhaustive search and ultimately heuristics are necessary.

Both the algorithms of Kruskal [1956] and Prim [1957] can be adapted to give constrained heuristic solutions although superior results are obtained via the algorithm of Esau and Williams [1966] in which a 'trade-off' function is maximised in order to determine the best way to reconnect the network at each stage. All three, however, can be shown to be variants of a single method (Kershenbaum & Chou [1974]). Further heuristics are offered by Frank et al. [1971] and Elias & Ferguson [1974].

For networks of moderate size, the improved techniques of Karnaugh [1976] and Kershenbaum et al. [1980] may be suitable. These are more accurate algorithms which will take longer to execute than the simple method above. Karnaugh's method, for example, makes repeated calls to a modified Esau Williams algorithm and produces results which are 2-3% better than those given otherwise.

The work carried out by Tang et al. [1978] and Hoc [1982] also deals with the choice of links in addition to the selection of concentrators to which the papers are more earnestly directed.

3.2. Generalised Networks.

This section deals with the more general case of section 3.1. This involves a number of nodes and switches in which traffic may flow from any node to any other node in the network. The situation is clearly far more complicated and, not surprisingly, fewer real algorithms exist for its solution.

In contrast, however, there is no shortage of literature concerning the problems involved. Gallego [1971], Frank & Chou [1972], Goldstein [1973], Engvall [1979] and Greenhop & Campbell [1984] all address themselves to the matter in various forms. The American ARPA network, in particular, has been studied in great detail (Frank et al. [1970], Cole [1972], Frank et al. [1972] and Shwartz [1977]).

Whilst providing powerful and useful insights into the behavior of operational communication networks, much of the work carried out to date does not allow genuine optimisation to be performed on a general network problem. It is certainly difficult, and probably impossible, to represent the complexities of a typical network problem by a mathematical model of a suitable form to allow conventional problem solving techniques to operate. For example, while linear programming problems are relatively easy to solve (Karmarkar [1984]), to express a network optimisation problem in such a form requires significant simplification in most areas. The results obtained via such an approach should thus be treated with extreme caution.

The natural alternative to modelling techniques, the systematic generation and evaluation of all possible solutions, is, whilst accurate, far too complex a task to consider for all but very small problems. The number of possible choices of M tandems from n nodes, for example, is ${}^{n}C_{M}$ so that the total number of valid sets is $\Sigma_{M=1..n}({}^{n}C_{M})$ and this takes no account of link placement or traffic routeing.

As before, heuristics are clearly needed to deal with large problems. The Double Drop method (BP [1984]) and the method of Grout et al. [1986,1987] mentioned in section 2.2 for determining tandem locations also provide methods of approximating the optimum link structure, the method of Grout et al. being a generalisation of the

Double Drop method. As before, it may be expected that the future will yield further heuristics for improving the accuracy of optimisation of the larger network problems. At present such techniques are extremely rare.

4. TRAFFIC ROUTEING STRATEGIES.

Suppose now that the topological (i.e. physical) structure of the solution network is determined. The locations of the nodes and switches are established and the appropriate links have been added between them. The result can be regarded as a graph on the n nodes. The solution is not complete, however. For a given node pair (i, j), there will be, in general, a number of paths from i to j. The problem of finding the best path for each node pair (remembering that each will interact) forms the basis of all traffic routeing algorithms. The problem, however, can manifest itself in a number of forms.

4.1. Shortest Path Problems.

The simplest form of routeing problem concerns finding the shortest path between i and j for each (i, j) in the network. It is simpler to merely consider two nodes at a time and to repeat the process as necessary. Suppose then that the shortest path between nodes u and v is required and that distances between all pairs of nodes in the network are known.

One of the first attempts at solving this problem is by Ford [1956] although some work was done earlier by Heller [1953]. The method given produces the shortest path in all cases but is not particularly computationally efficient. Dijkstra [1959] suggests an alternative which is probably the simplest of all shortest path algorithms.

Dijkstra's algorithm operates by including each node in turn into a set of nodes whose distance from u is known. Initially only u is in this set. At each subsequent stage the closest node to a node already in the set is chosen and its distance from u calculated as the minimum of its distance from w plus the distance of w from u for all w already in the set. The process continues until v becomes included in the set in this way. Dijkstra's algorithm is an improvement over Ford's because only a relatively small amount of work has to be done at each stage both in terms of computation and storage requirements.

Golden [1976] notes that an algorithm due to Bellman [1958] is more computationally efficient than Dijkstra's for sparse graphs with Euclidean distances and Golden & Ball [1978] suggest an improved version of Dijkstra's algorithm which also performs better than the original for similar examples.

A comparable problem, that of finding the shortest path through a number of nodes, is considered by Verblunsky [1951] and Beardwood et al. [1959]. The problem then increases considerably in complexity, however, and more resembles the famous travelling salesman problem (Lawler et al. [1985]).

4.2. Maximum Flow Problems.

Another way in which a traffic routeing problem can be expressed is in terms of finding the maximum traffic flow in the network. Assuming that the network is currently designed to carry the required amount of traffic, an effective choice of routeing may lead to a surplus capacity in some areas which can be utilised sometime into the future. It may be of great importance then to route all traffic around the network so that the maximum flow is possible.

To date, the major part of the results achieved with respect to maximum flow problems has been theoretical although in some instances the progression to the practical case is not extremely hard. It is assumed that all links within the network have known and fixed capacities. For any two nodes, u and v, within the network, the problem of finding the maximum possible flow from from u to v is a well solved one.

Menger [1927] showed that the maximum number of separate paths between u and v is equal to the minimum number of links needed to disconnect u from v completely (Harary [1971]). This result can be extended (Wilson [1972]) to show that the maximum flow possible between u and v is equal to the minimum sum of the capacities of such a set of links. Based upon this observation, Ford & Fulkerson [1956] propose an algorithm for finding such a flow based on a sequence of gradual improvements applied to a starting solution. The problem of calculating the magnitude of such a maximum flow with practical restrictions upon the length of each path, however, is known to be hard (Itai et al. [1982]). A heuristic procedure for this purpose is proposed by Ronen & Peri [1984].

The real problem, of course, is far more involved than this. Instead of traffic flowing between two nodes in isolation, all pairs of nodes in the network may need to communicate. Traffic between one pair may share links and switches with that between another pair (the capacity of the switches is not even considered by the simple algorithms although it can be incorporated into the capacity of the links with care). This problem has barely been approached in the literature on network optimisation and may well provide fruit if some work was to be directed at it.

4.3. General Routeing Problems.

The original and most common problem, however, is that of minimising the cost of the network. This involves the problem of routeing the traffic, not so as to minimise distance or maximise flow arbitrarily, but to find the most economical strategy for whatever form of cost is being used. Not surprisingly, the problem is far harder than the previous two and not particularly well solved in the practical case although there are some good examples of theoretical work available (Frank & Chou [1971], Gallager [1977], Yu et al. [1984], Esfahanian & Hakimi [1985] and Mars & Narendra [1987]). Many of these, however, attempt to restrict congestion etc. as opposed to minmising cost. Mulvey [1978] deals with a simplification of the minimum cost flow problem using the simplex linear programming method.

The real problem, however, is so much more difficult due, as much as any thing to the number of restrictions which may in force as well as the sheer number of routeings that are possible. Even if traffic is only permitted to pass via a maximum of one other node or tandem, there are of the order of n^{n^2} possible overall constructions (see Grout [1988]).

Again, practical techniques such as the double drop method (BP [1984]) and the method of Grout et al. [1986,1987] can achieve heuristic results to such problems but their accuracy in this case is not entirely proven. This is possibly one area of optimisation, if any, which urgently needs a considerable improvement in the availability of practical results and algorithms.

5. CONCLUSIONS.

Clearly there is no shortage of literature available concerning network optimisation in all its forms. In particular, the simplifications of the general case appear to be well solved. The more general problem, however, is still proving difficult and may well remain so for some time (see Greenhop & Campbell [1984] or Grout [1988] for further discussion).

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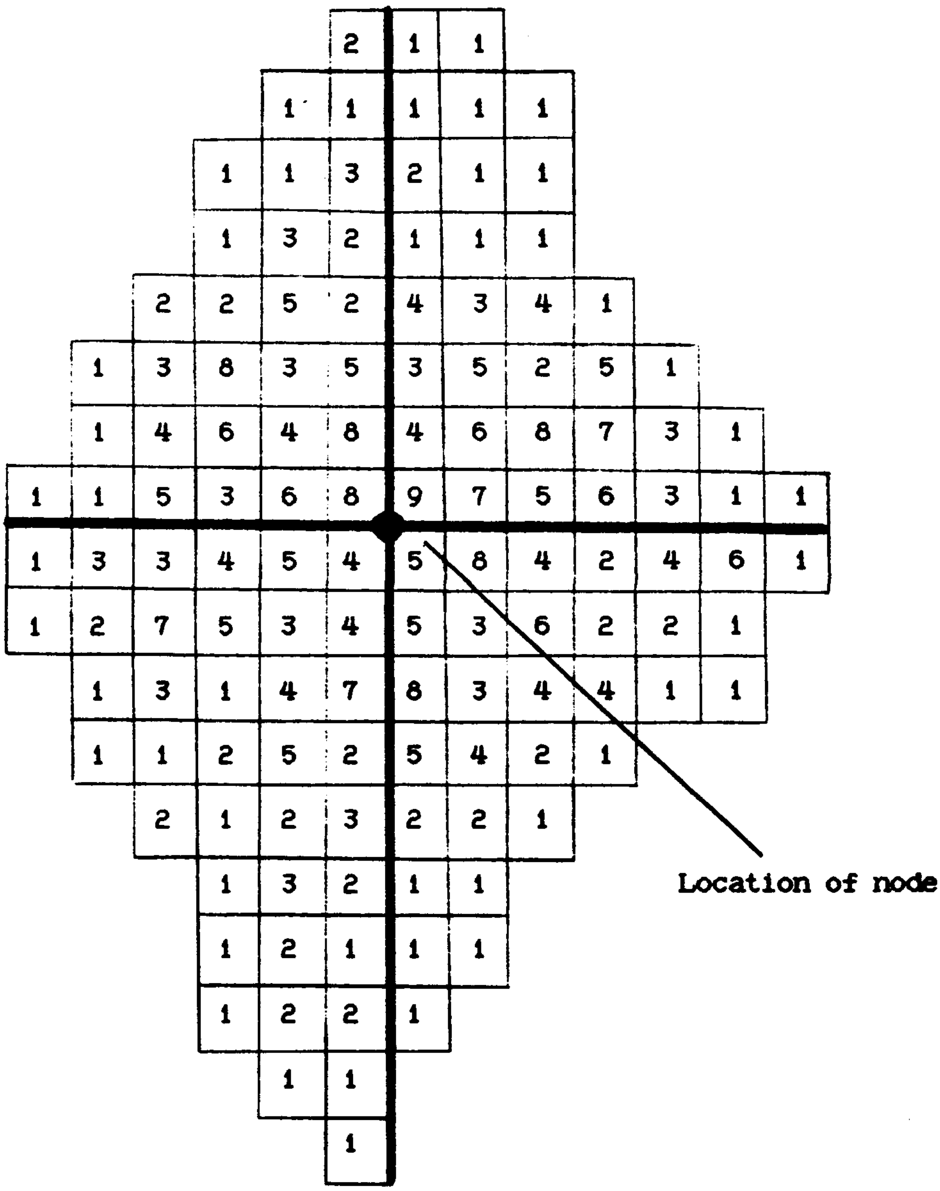

Figure 1. Subscriber Density.

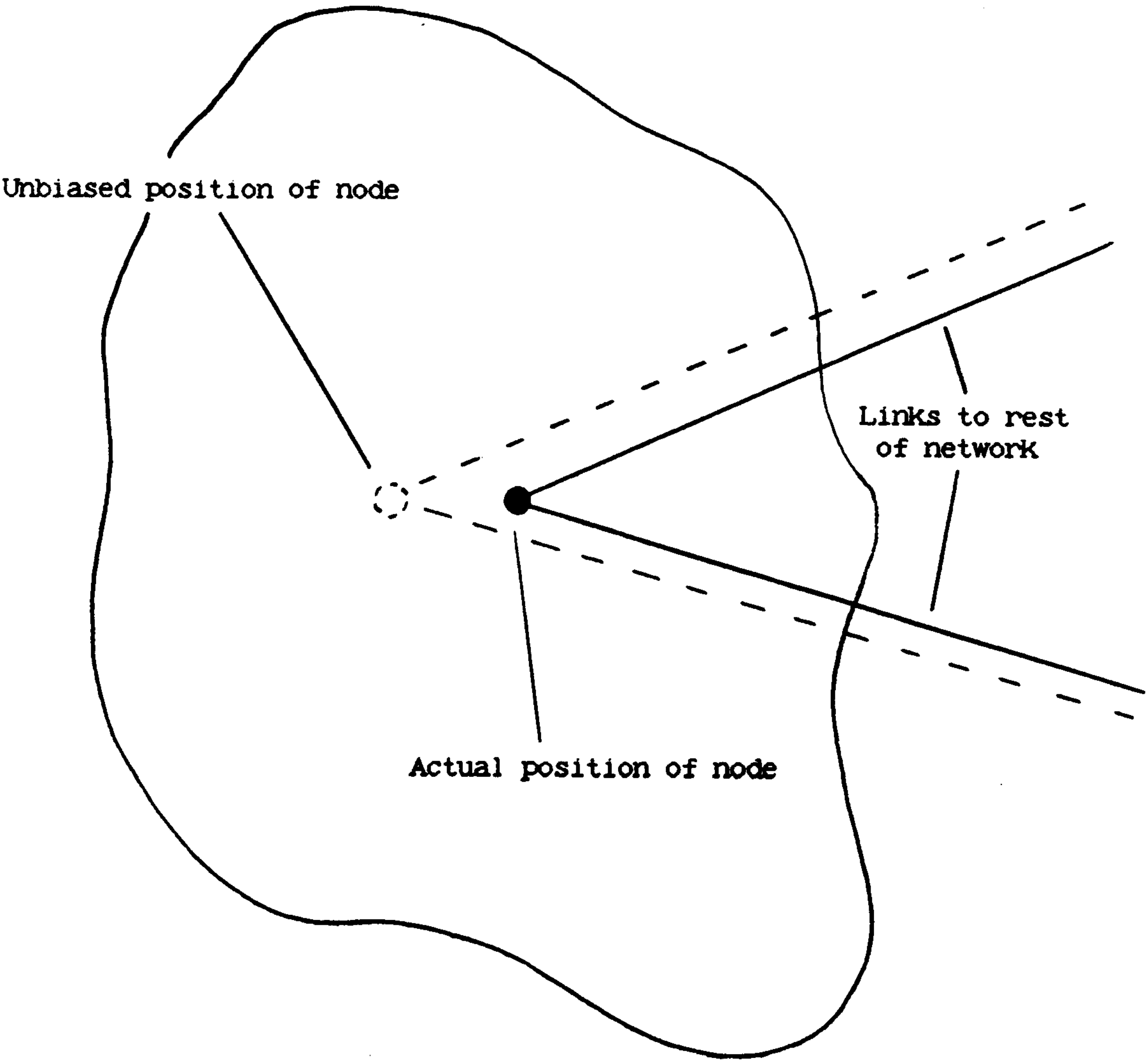

Figure 2. Node Location.

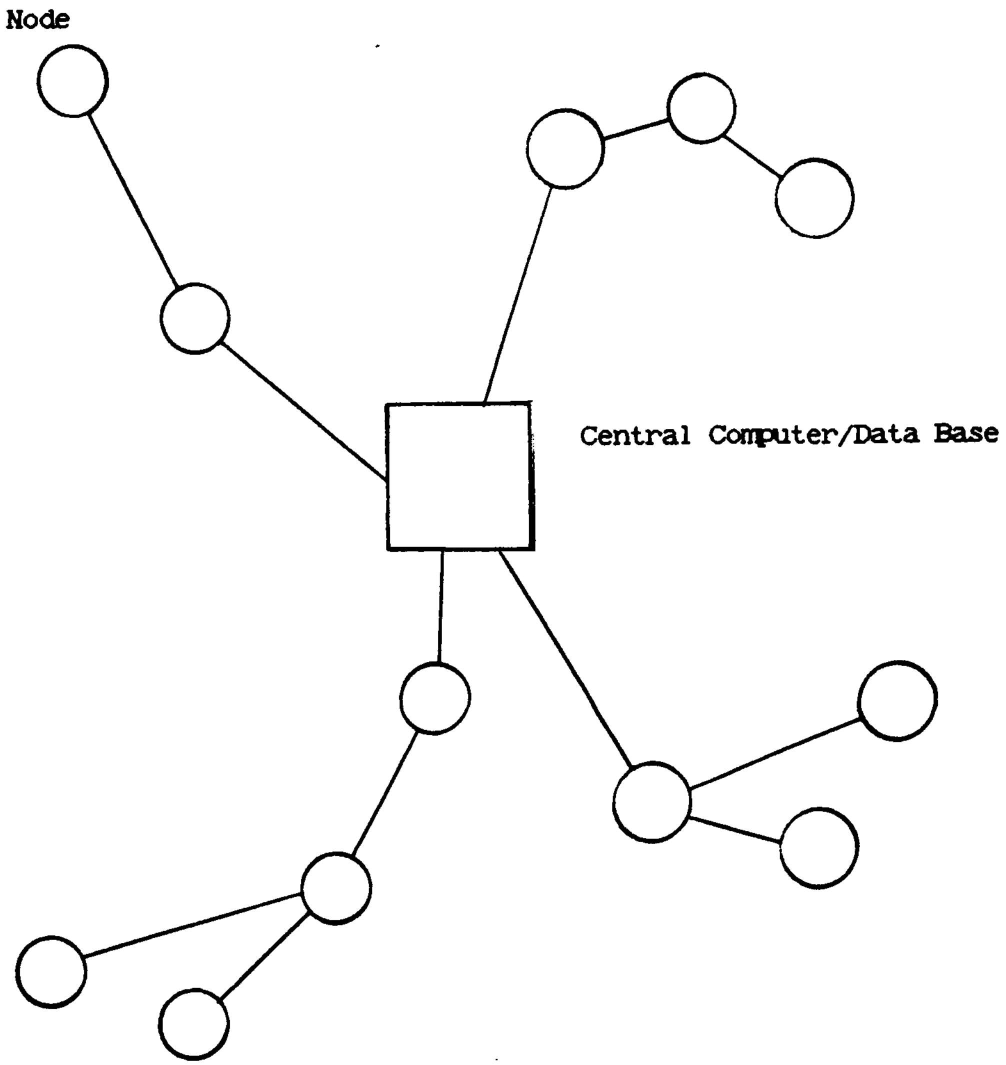

Figure 3. Centralised Network.